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Calculus

No. :

Assignment 4

1. $A = \begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}$

* $(\lambda I - A)x = 0$

$$\left(\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}\right)x = 0$$

$$\begin{pmatrix} \lambda & -3 \\ -1 & \lambda-2 \end{pmatrix}x = 0$$

$\det(\lambda I - A) = 0$

$\det\begin{pmatrix} \lambda & -3 \\ 1 & \lambda-2 \end{pmatrix} = 0$

$(\lambda)(\lambda-2) - (3) = 0$

$\lambda^2 - 2\lambda - 3 = 0$

$(\lambda-3)(\lambda+1) = 0$

$\lambda = 3 \vee \lambda = -1$

* Vektor λ

$(\lambda I - A)x = 0$

$$\begin{pmatrix} \lambda & -3 \\ -1 & \lambda-2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

⊖ $\lambda = 3$

$$\begin{pmatrix} 3 & -3 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$3x_1 - 3x_2 = 0$

$-x_1 + x_2 = 0$

$x_1 = 1$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$x_2 = 1$

⊖ $\lambda = -1$

$$\begin{pmatrix} -1 & -3 \\ -1 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$-x_1 - 3x_2 = 0$

$-x_1 - 3x_2 = 0$

$x_1 = 3$ $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$x_2 = -1$

* $P = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$

$$P^{-1} = \frac{1}{3+1}\begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1/4 & -1/4 \\ 1/4 & 3/4 \end{pmatrix}$$

* $P^{-1}AP$

$$= \begin{pmatrix} 1/4 & -1/4 \\ 1/4 & 3/4 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/4 & -1/4 \\ 1/4 & 3/4 \end{pmatrix}\begin{pmatrix} -3 & 3 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -3/4 - 1/4 & 3/4 - 3/4 \\ -3/4 + 3/4 & 3/4 + 9/4 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}$$

2. $A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

* $|A - \lambda I| = 0$

$$\left| \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\left| \begin{array}{ccc|cc} 3-\lambda & -2 & 0 & 3-\lambda & -2 \\ 2 & 3-\lambda & 0 & -2 & 3-\lambda \\ 0 & 0 & 5-\lambda & 0 & 0 \end{array} \right. = 0$$

$(3-\lambda)(3-\lambda)(5-\lambda) - (5-\lambda).4 = 0$

$45 - 35\lambda + 11\lambda^2 - \lambda^3 - (20 - 4\lambda) = 0$

$25 - 35\lambda + 11\lambda^2 - \lambda^3 = 0$

$-\lambda^3 + 11 - 35\lambda + 25 = 0$

$$\begin{array}{c|cccc} & -1 & 11 & -35 & 25 \\ 5 & & -5 & 30 & -25 \\ \hline & -1 & 6 & -5 & 0 \end{array}$$

$(\lambda - 5)(-\lambda^2 + 6\lambda - 5) = 0$

$(\lambda - 5)(-\lambda + 1)(\lambda - 5) = 0$

$\lambda = 5 \quad \vee \quad \lambda = 1$

* $P = \begin{pmatrix} -1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ * $P^{-1} = \begin{pmatrix} -1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}$

* $P^{-1} A P$

$$= \begin{pmatrix} -1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix} \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -5/2 & 5/2 & 0 \\ 0 & 0 & 5 \\ 1/2 & 1/2 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix}}}$$